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B.A./B.Sc. III Semester Examination

$CBS - IIIs/1(R\&P)$

Mathematics
Course No.: UMTTC301

Time Allowed: 3 Hours **Maximum Marks: 80**

Note: Attempt any four questions from the given questions. Each question carries equal marks.

Q 1. *i) Define countable set. Prove that the set of all sequences whose elements are either zero or one is not countable.*

ii) Prove that there exist infinite number of irrational numbers between 0 *and* 0.01.

Q 2. *i) Suppose that* $A = \{sinx - 2cosx \colon x \in \mathbb{R}\}$ *and* $B = \{\frac{1}{1+x^2} \colon 0 \leq x \leq 1\}$. *Find l.u.b and g.l.b of* $3A + 2B$.

ii) Prove that monotonically increasing sequence $\{a_n\}$ *converges iff it is bounded above.*

Q 3. *i) State and prove Cauchy's general principle of convergence of a sequence.*

ii) Show that $\{a^n\}$ *converges to* 0 *if* $-1 < a < 1$.

Q 4. *i) Show that if the series* $\sum a_n$ *is convergent, then* a_n *converges to* 0. *Hence show that the series* $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + ...$ *is not convergent.*

ii) State and prove Alternating Series Test.

Q 5. *i) Discuss the convergence or divergence of the series*

$$\sum (\sqrt{n^4 + 1} - \sqrt{n^4 - 1})$$

ii) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(logn)^p}$, $p > 0$ *converges for* $p > 1$ *and diverges for* $p \leq 1$.

Q 6. *i) Let* $f\colon [a,\ b] \to \mathbb{R}$ *be a continuous function and* $f(a) \neq f(b)$*. Prove that* f *assumes every value between* $f(a)$ *and* $f(b)$*.*

ii) Show that every continuous function $f\colon [a,\ b] \to \mathbb{R}$ *is uniformly continuous.*

Q 7. *i) Prove that every continuous function* $f\colon [a,\ b] \to \mathbb{R}$ *is bounded.*

ii) Let f *be continuous on an interval* I *containing* a *and* b*. If* λ *lies between* $f(a)$ *and* $f(b)$*, show that there exists a real number* c *between* a *and* b *such that* $\lambda = f(c)$*.*

Q 8. *i) Define pointwise convergence and uniform convergence of a sequence of functions. Give an example of a sequence of functions which is pointwise convergent but not uniformly convergent.*

ii) State M_n-*test for uniform convergence. Use* M_n-*test to show that the sequence* $\{f_n(x)\}$*, where* $f_n(x) = \frac{nx}{1+n^2x^2}$ *is not uniformly convergent on any interval containing* 0*.*